Eur. Phys. J. B **9**, 201–206 (1999)

THE EUROPEAN PHYSICAL JOURNAL B
EDP Sciences

Normal and superconducting state in the presence of forward electron-phonon and impurity scattering

O.V. Danylenko[1,2], O.V. Dolgov[1,2,a], M.L. Kulić[3], and V. Oudovenko[4,5]

[1] P.N. Lebedev Physical Institute, Moscow, Russia
[2] Institut für Theoretische Physik, Universität Tübingen, Germany
[3] Physikalisches Institut, Universität Bayreuth, 95440 Bayreuth, Germany
[4] Max-Planck-Institut für Festkörperforschung, Heisenbergstrasse 1, 70569 Stuttgart, Germany
[5] Joint Institute for Nuclear Research, 141980 Dubna, Russia

Received 1st July 1998

Abstract. By assuming that the superconducting pairing is due to the forward *E-P* scattering (*FEP* pairing) it is shown that the critical temperature of clean systems T_{c0} depends linearly on the *E-P* coupling constant λ and the isotope effect α is very small. Impurities with the pronounced forward scattering (*FS* impurities) change analytical properties of the quasiparticle Green's function substantially compared to the case of the isotropic scattering. The *FS* impurities are pair-breaking and affect in the same way *s*- and *d-wave FEP* pairing making $\alpha = 1/2$ in the dirty limit.The usual isotropic impurity scattering is pair-weakening for *s-wave* and pair-breaking for the *d-wave FEP* pairing.

PACS. 74.20.-z Theories and models of superconducting state – 74.25.-q General properties; correlations between physical properties in normal and superconducting states

1 Introduction

There are growing experimental evidences for *d-wave* pairing in high-T_c superconductors (HTS) [1] – seemingly in contradiction with the standard phonon mechanism. The search for the pairing mechanism in HTS materials has opened new directions in the theory of superconductivity. For instance, in [2,3] it is proposed the antiferromagnetic spin-fluctuation (AFS) pairing in copper-oxides with pronounced peaks at $\mathbf{Q} = (\pm\pi, \pm\pi)$ in the spin-fluctuation spectral density $P_s(\mathbf{k}, \omega)$. Since the treatment of the AFS pairing is approximate and uncontrollable, it is still unclear which mechanism is underlying *d-wave* pairing in HTS systems. However, a possibility for the phonon mechanism of superconductivity in HTS materials has been analyzed in [4,5], where the strong *E-P* coupling was extracted from an analysis of optic [6] and tunneling measurements [7]. In that respect one expects that the very sophisticated and recently developed methods of synthesis of HTS oxides [8] will give an impetus for new tunneling measurements, which might resolve the role of the *E-P* interaction in the pairing mechanism.

The possibility of *d-wave* pairing in HTS materials due to the renormalized (by strong electronic correlations) *E-P* coupling has been put forward in a series of papers [9], where it was shown that for small hole doping δ strong Coulomb correlations renormalize the *E-P* interaction giving rise to the pronounced forward (small-$\mathbf{q}$) scattering peak, while the backward scattering is strongly suppressed. This renormalization of the square of the *E-P* coupling constant $|g(\mathbf{q})|^2$ is described by the vertex function $\Gamma(\mathbf{q})$, *i.e.* $|g_{\rm scr}(\mathbf{q})|^2 = |g_0(\mathbf{q})|^2\Gamma^2(\mathbf{q})$, where $g_0(\mathbf{q})$ is the bare coupling constant and $\Gamma^2(\mathbf{q})$ is strongly peaked at $\mathbf{q} = 0$. Note, in what follows we assume that the square of $\Gamma(\mathbf{q})$ and therefore the square of the *E-P* coupling constant are strongly peaked at $\mathbf{q} = 0$ and has the delta-function singularity, *i.e.* $|g_{\rm scr}(\mathbf{q})|^2 \approx |g_0(\mathbf{q})|^2\delta(\mathbf{q})$. This specific screening of the *E-P* interaction suppresses electric resistivity [9,10] and can also lead to *d-wave* superconductivity [9]. Its physical meaning is that each quasiparticle, due to the suppression of the doubly occupancy on the same lattice, is surrounded by a giant correlation hole with the characteristic size $R \simeq a/\delta$, where a is the lattice constant. These results are confirmed later on by the slave-boson method [11]. The long-range *E-P* interaction (pronounced forward scattering) can be also due to the poor Coulomb screening in HTS oxides, which has been proposed and elaborated in [12,13]. The forward scattering might be pronounced in case of a large density of states near some $\mathbf{k}$-points at the Fermi surface [14]. The possibility of the forward scattering and the long-range forces between quasiparticles in strongly-interacting low-dimensional systems was analyzed by Anderson and coworkers [15]. They anticipated the failure of the Fermi-liquid theory for the 2-D Hubbard model due

[a] e-mail: oleg.dolgov@uni-tuebingen.de

to the orthogonality catastrophe. In [14] it is shown that in the presence of the pronounced forward *E-P* interaction (*FEP* pairing) the vertex (non-Migdal) corrections might increase the superconducting critical temperature T_c. However, in [14] only small angle scattering was considered, which is different from the scattering pronounced at small $\mathbf{q}$ considered here and in [9,12,13]. Recently it was argued [16] that the forward *E-P* scattering can, in combination with the topology of the electronic Fermi surface in HTS oxides, even produce linear T dependent resistivity down to temperatures as 10 K. The nonmagnetic impurity scattering is also renormalized by strong correlations in the similar manner as the *E-P* interaction [9], *i.e.* $u_{\rm scr}(\mathbf{q}) = u_0(\mathbf{q})\Gamma(\mathbf{q})$, where $u_0(\mathbf{q})$ is the bare scattering potential. We stress here that in the paper the delta-function singularity for the square of the impurity potential is assumed, *i.e.* $u^2_{\rm scr}(\mathbf{q}) \approx u^2(\mathbf{q})\delta(\mathbf{q})$ – which is called the forward impurity scattering (*FS* impurities). The pronounced forward scattering in the pairing interaction and in the impurity scattering causes substantial change of change the physics of the problem as it is demonstrated below. Note, the problem of the pronounced forward scattering is very delicate and far from understanding, as it has been already discussed in the framework of the phenomenological Landau's Fermi liquid [17].

In the following analysis we assume, as we already said, an extreme case of the forward electron-phonon interaction (*FEP* pairing) and of the forward nonmagnetic impurity scattering (*FS* impurities), *i.e.* that $|g_{\rm scr}(\mathbf{q})|^2 \sim \delta(\mathbf{q})$ and $u^2_{\rm scr}(\mathbf{q}) \sim \delta(\mathbf{q})$, where $\delta(\mathbf{q})$ is the Dirac delta-function. We emphasize that this rather extreme approximation (for long-range forces) picks up the main physics, and at the same time it is a valuable approximation whenever the range R of the effective interaction fulfills the condition $R \gg k_{\rm F}^{-1}$, where the momentum cut-off in the pairing potential q_c is very small, *i.e.* $q_c(\sim 1/R) \ll k_{\rm F}$. The finite cut-off effects are also studied in this paper, where it is shown that the *delta-function approximation* is the leading order with respect to the small parameter $q_c/k_{\rm F}$. Moreover, the delta-function approximation greatly simplifies the structure of the Eliashberg equations by omitting integration in k-space. A similar approximation is used for the AF spin-fluctuation mechanism of pairing, where four peaks at $Q = (\pm\pi, \pm\pi)$ in the spin-fluctuation density $P_{\rm sf}(k,\omega)$ were replaced by four delta-functions [18]. The constructive and destructive interplay of the *FEP* and A*FS* mechanisms in the case of *d-wave* pairing was studied in [19].

In this paper we show: (*i*) that the superconducting critical temperature T_c for the *FEP* pairing deviates substantially from the BCS formula; (*ii*) the *FS* impurities induce substantial self-energy effects in the normal state and affect T_c for *d*- and *s-wave* *FEP* pairing equally, while they do not affect the usual isotropic BCS pairing. Vertex corrections for the *FS* impurities in the normal state are studied in the ladder approximation, while the more complicated cases of the *E-P* and impurity vertex corrections in the normal and superconducting state is, due to complexity, a matter of future activity.

2 Eliashberg equations for FEP pairing and FS impurities

Let us write the Eliashberg equations in the presence of the *FEP* pairing potential $V_{\rm ep}(\mathbf{k},\omega) = \delta(\mathbf{k})V_{\rm ep}(\omega)$ and in the presence of the *FS* impurities ($u^2_{\rm scr}(\mathbf{k}) = \delta(\mathbf{k})u^2$), where the latter is treated first in the self-consistent Born approximation – on vertex corrections see below. The normal and anomalous Green's functions are defined by $G(\mathbf{k},\omega_n) = -[{\rm i}\omega_n Z(\mathbf{k},n) + \bar{\xi}_n(\mathbf{k})]/D(\mathbf{k},n)$ and $F(\mathbf{k},\omega_n) = -Z(\mathbf{k},n)\Delta(\mathbf{k},n)/D(\mathbf{k},m)$ respectively. The renormalization function $Z(\mathbf{k},n) \equiv Z_n(\xi)$, the energy renormalization $\bar{\xi}(\mathbf{k},n) \equiv \bar{\xi}_n(\xi)$ and the superconducting order parameter $\Delta(\mathbf{k},n) \equiv \Delta_n(\xi)$ are solutions of the following equations ($\omega_n = \pi T(2n+1)$)

$$Z_n(\xi) = 1 + \frac{T}{\omega_n}\sum_m V_{\rm eff}(n-m)\frac{\omega_m Z_m(\xi)}{D_m(\xi)},$$
$$\bar{\xi}_n(\xi) = \xi(\mathbf{k}) - T\sum_m \frac{V_{\rm eff}(n-m)}{D_m(\xi)}\bar{\xi}_m(\xi),$$
$$Z_n(\xi)\Delta_n(\xi) = T\sum_m \frac{V_{\rm eff}(n-m)Z_m(\xi)\Delta_m(\xi)}{D_m(\xi)}\,. \quad (1)$$

Here, $V_{\rm eff}(n-m) = V_{\rm ep}(n-m) + \delta_{n,m} n_i u^2/T$ and $D_n(\xi) = [\omega_n Z_n(\xi)]^2 + \bar{\xi}^2_n(\xi) + [Z_n(\xi)\Delta_n(\xi)]^2$, $\xi(\mathbf{k})$ is the bare quasiparticle spectrum and n_i is the impurity concentration.

3 Normal state in the presence of FS impurities only

Let us consider the normal state and neglect for the moment the *E-P* interaction, *i.e.* we put $V_{\rm ep}(n-m) = 0$ and $\Delta_n(\xi) = 0$ and look for the renormalization of the Green's function by the *FS* impurities in the self consistent Born approximation, *i.e.*

$$G^{-1}(\mathbf{k},\omega_n) \equiv {\rm i}\bar{\omega}_n(\xi) - \bar{\xi}_n(\xi) = G_0^{-1}(\mathbf{k},\omega_n) - \Sigma^{\rm imp}_{\rm B}(\mathbf{k},\omega_n),$$

where ${\rm i}\bar{\omega}_n(\xi) \equiv {\rm i}\omega_n Z_n(\xi)$. The self-energy in this approximation is given by

$$\Sigma^{\rm imp}_{\rm B}(\mathbf{k},\omega_n) = \Gamma^2_{\rm F} G(\mathbf{k},\omega_n) \quad (2)$$

where $\Gamma_{\rm F} = \sqrt{n_i}u$ – see Figure 1a. The solutions for $\bar{\xi}_n$ and $\bar{\omega}_n$ are

$$\bar{\xi}_n = \xi\left[\frac{1}{2} + \frac{\omega_n}{\sqrt{(\omega_n + {\rm i}\xi)^2 + 4\Gamma^2_{\rm F}} + \sqrt{(\omega_n - {\rm i}\xi)^2 + 4\Gamma^2_{\rm F}}}\right],$$
$$\bar{\omega}_n = \frac{\omega_n}{2} + \frac{1}{4}\left[\sqrt{(\omega_n + {\rm i}\xi)^2 + 4\Gamma^2_{\rm F}} + \sqrt{(\omega_n - {\rm i}\xi)^2 + 4\Gamma^2_{\rm F}}\right]. \quad (3)$$

By using equation (3) and for the standard isotropic spectrum one obtains for the density of states $N(\omega)/N(0) = 1$,

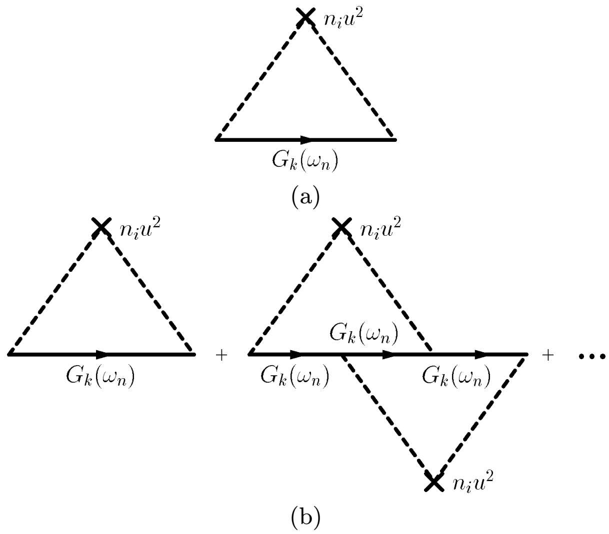

Fig. 1. (a) The self-energy $\Sigma_{\mathrm{B}}^{\mathrm{imp}}(\mathbf{k},\omega_n) = \Gamma_{\mathrm{F}}^2 G(\mathbf{k},\omega_n)$ in the presence of the FS impurities in the Born approximation; (b) the self-energy $\Sigma_{\mathrm{V}}^{\mathrm{imp}}(\mathbf{k},\omega_n) = \Gamma_{\mathrm{F}}^2 G(\mathbf{k},\omega_n)/[1-\Gamma_{\mathrm{F}}^2 G^2(\mathbf{k},\omega_n)]$ in the presence of the FS impurities with ladder vertex corrections.

i.e. $N(\omega)$ is *unrenormalized* in the presence of the FS impurities – like in the case of the normal isotropic impurity scattering (NS impurities). This might be not the case for an anisotropic spectrum. However, the FS impurities strongly renormalize the inverse life-time $\tau_{\mathrm{B}}^{-1}(\omega) \equiv -\mathrm{Im}\Sigma_{\mathrm{B}}^{\mathrm{imp}}(\xi=0,\omega)$. By using equation (3) and by the analytical continuation of $\Sigma_{\mathrm{B}}^{\mathrm{imp}}(\xi=0,\mathrm{i}\omega_n \to \omega+\mathrm{i}\delta)$ one obtains ($\Gamma_F = \sqrt{n_i}u$)

$$-\mathrm{Im}\Sigma_{\mathrm{B}}^{\mathrm{imp}}(\xi=0,\omega) = \Gamma_{\mathrm{F}}\Theta(2\Gamma_{\mathrm{F}}-\omega)\sqrt{1-\left(\frac{\omega}{2\Gamma_{\mathrm{F}}}\right)^2}. \quad (4)$$

Θ is the Heavyside function. We see that the FS impurities (in the self-consistent Born approximation) introduce a nonanalyticity of $\tau_{\mathrm{B}}^{-1}(\omega=0)$ as a function of n_i, *i.e.* $\tau_{\mathrm{B}}^{-1}(\omega=0) = \sqrt{n_i}u$. Note, for the NS impurities one has $\tau_{\mathrm{B}}^{-1}(\omega=0) \sim n_i N(0) u^2$.

Due to the absence of integration over momenta in $\Sigma^{\mathrm{imp}}(\mathbf{k},\omega_n)$ for the FS impurities there is no small parameter in the theory, like $k_{\mathrm{F}}l$ for the NS impurities, where k_{F} is the Fermi wave vector and l is the quasiparticle mean-free path. Therefore, the vertex corrections for the FS impurities become important – see Figure 1b. The summation of the *ladder* diagrams in Figure 1b yields the self-energy

$$\Sigma_{\mathrm{V}}^{\mathrm{imp}}(\mathbf{k},\omega_n) = \frac{\Gamma_{\mathrm{F}}^2 G(\mathbf{k},\omega_n)}{1-\Gamma_{\mathrm{F}}^2 G^2(\mathbf{k},\omega_n)}, \quad (5)$$

where $\Gamma_{\mathrm{F}} = \sqrt{n_i}u$. It is seen from equation (5) that the ladder vertex corrections screen impurity scattering in the Born approximation. The retarded quasiparticle Green's function $G(\xi=0,\mathrm{i}\omega_n \to \omega+\mathrm{i}\delta) \equiv G_{\mathrm{R}}(\omega)$ is given by

$$G_{\mathrm{R}}(\omega) = \frac{12^{1/6}}{\sqrt{3}\Gamma_{\mathrm{F}}}\left[\frac{\omega}{\Phi(\omega)} + \frac{1}{12^{1/3}}\frac{\Phi(\omega)}{\omega\Gamma_{\mathrm{F}}^2}\right], \quad (6)$$

where $\Phi(\omega) \equiv \Phi(\mathrm{i}\omega_n \to \omega+\mathrm{i}\delta)$ and

$$\Phi(\mathrm{i}\omega_n) = \left[9\Gamma_{\mathrm{F}}^4\omega_n^2 + \sqrt{(9\Gamma_{\mathrm{F}}^4\omega_n^2)^2 + 12\Gamma_{\mathrm{F}}^6\omega_n^6}\right]^{1/3}. \quad (7)$$

In the limit of small frequency, *i.e.* for $\omega \ll 3\sqrt{3}\Gamma_{\mathrm{F}}/2$ the imaginary part of the self-energy (the inverse life-time $\tau_{\mathrm{V}}^{-1} \equiv -\mathrm{Im}\Sigma_{\mathrm{V}}^{\mathrm{imp}}(\xi=0,\omega)$) has the form

$$-\mathrm{Im}\Sigma_{\mathrm{V}}^{\mathrm{imp}}(\xi=0,\omega) \approx \Gamma_{\mathrm{F}}^{2/3}\omega^{1/3}. \quad (8)$$

Note that $\tau_{\mathrm{V}}^{-1} \sim \omega^{1/3}$, which means that $\tau_{\mathrm{V}}^{-1} \ll \tau_{\mathrm{B}}^{-1}$ for $\omega \ll \Gamma_{\mathrm{F}}$, *i.e.* the scattering by many impurities, calculated in the ladder vertex approximation, screen the single impurity Born scattering. The density of states in the vertex approximation will be studied elsewhere [20].

4 Superconductivity due to FEP and in the presence of FS impurities

In further analysis it is assumed that the pairing is due to the forward electron-phonon scattering – the FEP pairing. Therefore, one expects in this case that T_{c} is different from the usual BCS (or Eliashberg) formula and that the FS and NS impurities affect this pairing strongly. In further analysis of superconductivity due to FEP mechanism two assumptions are made: (1) the *E-P* interaction is considered in the weak coupling limit – the strong coupling limit will be studied elsewhere [20]; (2) the effects of the FS impurities on T_{c} are studied in the self-consistent Born approximation and vertex corrections are not taken into account, because anomalous vertex corrections to the gap equation are at present unknown. However, the effects of the FS impurities, treated in the Born approximation, on T_{c} are expected to be preserved qualitatively if the anomalous vertex corrections are included. In the latter case one expects less reduction of T_{c}, due to the screening by the vertex corrections.

4.1 T_{c} due to FEP pairing in clean systems

We find T_{c} in the weak coupling limit [21], where $V_{\mathrm{ep}}(n-m) \approx V_{\mathrm{ep}}\Theta(\Omega-|\omega_n|)\Theta(\Omega-|\omega_m|)$ and Ω is the phonon cut-off energy. In this limit one obtains $Z(\mathbf{k},n)=1$ and the FEP pairing gives the maximum T_{c} on the Fermi surface, *i.e.* for $\xi=0$ where $\bar{\xi}_n(\mathbf{k},n)=0$. The solution of the equation (1) in the weak coupling limit and for $T_{\mathrm{c}} \ll \Omega$ is given by

$$T_{\mathrm{c0}} = \frac{\lambda}{4N(0)} = \frac{V_{\mathrm{ep}}}{4}\,. \quad (9)$$

Several points should be stressed. First, T_{c0} depends linearly on $\lambda = N(0)V_{ep}$, which is due to the delta-function form of $V_{ep}(\mathbf{k},\omega)$. This result is similar to that obtained in [18] for the AFS pairing, where the delta-function limit was used [18]. Second, contrary to the FEP pairing there is a threshold condition for the pairing potential $V_{sp} > 2|\mu|$ in the case of the AFS pairing, where V_{sp} is the spin-fluctuation constant and μ is the chemical potential [18]. Third, in the case of the pairing potential $V_{ep}(\mathbf{q},\omega_n)$ with the finite cut-off $q_c \neq 0$ it is shown [20] that $T_{c0} \equiv T_{c0}(q_c = 0)$ is the zeroth-order with respect to q_c. For the short-range pairing potential with $q_c \sim 2k_F$, *i.e.* when $q_c V_F \sim W \sim 1/N(0)$ one obtains the standard BCS result $T_{c0}^{BCS} = 1.13\Omega\exp(-1/\lambda)$, while for the long-range pairing potential with the cut-off $q_c V_F \ll \Omega$ (the FEP pairing) the finite-q_c correction to T_{c0} is given by

$$T_c \simeq T_{c0}\left(1 - \frac{7\zeta(3) q_c V_F}{4\pi^2 T_{c0}}\right). \tag{10}$$

The finite value of q_c lowers T_{c0} of the FEP ($q_c = 0$) pairing. Fourth, in the weak coupling limit there is no isotope effect, *i.e.* $(-\mathrm{d}\ln T_{c0}/\mathrm{d}\ln M) = 0$ for $\Omega \to \infty$ and $T_{c0} \ll \Omega$, although the pairing is due to the E-P interaction! Note, that small α is obtained also in the case $q_c \ll k_F$. The strong coupling (retardation) effects introduce a mass (M) dependence of T_c. For example, by assuming the single mode (Einstein) phonon spectrum with the frequency Ω one obtains $V_{eff}(\omega_n) = V_{ep}\Omega^2/(\Omega^2 + \omega_n^2)$ – see equation (1). In case when $[\lambda N^{-1}(0)/2\Omega] < 1$ one obtains

$$Z_n(0) \simeq 1 + \frac{\lambda N^{-1}(0)}{2\Omega}\frac{\Omega^2}{\Omega^2 + \omega_n^2}\cdot \tag{11}$$

Because $Z_n > 1$ (note $\bar{\xi}_n(\xi = 0) = 0$) the critical temperature

$$T_{c0}^{(1)} = \frac{T_{c0}}{[1 + \lambda/2\Omega N(0)]^2} \tag{12}$$

is decreased, *i.e.* $T_{c0}^{(1)} < T_{c0}$. Since $\Omega \sim M^{-1/2}$ one obtains $\alpha \equiv -(\mathrm{d}\ln T_{c0}^{(1)}/\mathrm{d}\ln M) \approx T_{c0}/\Omega \ll 1/2$ ($a \sim 1$), because $T_{c0} \ll \Omega$ is assumed. The problem of the isotope effect is much more complicated than it is treated here. However, this simplified analysis shows a connection between small α and the dominance of small-q scattering in the pairing potential. This interesting result is an impetus for future study.

4.2 T_c due to FEP in the presence of FS impurities

The FS impurities affect T_c, which is due to the FEP pairing. In the weak coupling limit T_c is given by

$$1 = V_{ep}T_c\sum_{\omega_n=-\Omega}^{\Omega}\frac{1}{\omega_n^2 Z_n(\xi = 0)}, \tag{13}$$

where $Z_n(\xi = 0) = (1 + \sqrt{1 + 4\Gamma_F^2/\omega_n^2})/2$ and $\bar{\xi}_n(\xi = 0) = 0$. Note, equation (13) holds for all kind of FEP pairings (s-, d-, etc.). Some limiting cases are considered: (a) $\Gamma_F \ll \pi T_c$ – in that case T_c is given by

$$T_c \approx T_{c0}\left[1 - \frac{4\Gamma_F}{49T_{c0}}\right] \tag{14}$$

(b) $\Gamma_F \gg \pi T_c$ – if $\Gamma_F \gg \Omega/2$ is fulfilled one obtains

$$T_c \approx \frac{\pi\Omega}{2\gamma}\exp(-\pi\Gamma_F/V_{ep}), \quad \gamma \approx 1.78. \tag{15}$$

We point out two results. First, the FS impurities are *pair weakening* for the FEP pairing – the exponential fall-off of T_c with an increase of Γ_F. This is contrary to the pair breaking effect of the NS impurities on usual *d-wave* pairing, where $T_c = 0$ for $\Gamma_{cr} \approx 0.8T_{c0}$. Second, the FS impurities give rise to the large isotope effect $\alpha = 1/2$ in the dirty limit $\Gamma_F \gg T_c$ – note T_{c0} is mass independent.

4.3 T_c due to FEP in the presence of NS impurities

In this case one should make difference between *s-wave* and *d-wave* FEP pairings. For the NS impurities one has $\bar{\xi}_n(\xi) = 0$ and $Z_n = 1 + \Gamma/|\omega_n|$, where $\Gamma = \pi N(0)u^2$.

4.3.1 d-wave FEP pairing

In that case T_c in the weak coupling limit is given by

$$1 = V_{ep}T_c\sum_{\omega_n=-\Omega}^{\Omega}\frac{1}{\omega_n^2 Z_n^2}\cdot \tag{16}$$

Some limiting cases of equation (16) are interesting.

(a) For $\Gamma \ll \pi T_c$ one obtains

$$T_c \cong T_{c0}\left(1 - \frac{2\Gamma}{\pi T_{c0}}\right)\cdot \tag{17}$$

Note, for the *d-wave* FEP pairing the slope $(-\mathrm{d}T_c/\mathrm{d}\Gamma) = 2/\pi$ is smaller than the slope of the usual *d-wave* pairing in the presence of the NS impurities, where $(-\mathrm{d}T_c/\mathrm{d}\Gamma) = \pi/4$. As a consequence the *d-wave* FEP pairing is *more robust* in the presence of the NS impurities than the usual *d-wave* pairing [22];

(b) $\Gamma \gg \pi T_c$ – in this limit one obtains from equation (16) that $T_c = 0$ for $\Gamma_{cr} \simeq (4/\pi)T_{c0}$. Note, in the case of the usual *d-wave* pairing $\Gamma_{cr} \simeq 0.8T_{c0}$, which confirms our statement on the robustness of the *d-wave* FEP pairing, compared with the usual *d-wave* pairing.

4.3.2 s-wave FEP pairing

T_c in the weak coupling limit has the form

$$1 = V_{ep}T_c\sum_{\omega_n=-\Omega}^{\Omega}\frac{1}{\omega_n^2 Z_n}\cdot \tag{18}$$

The denominator in equation (18) is proportional to Z_n while for the *d-wave FEP* pairing to Z_n^2 – see equation (16). Since $Z_n > 1$ it means that in the presence of the NS impurities *s-wave FEP* pairing is *more robust* than *d-wave FEP* pairing. By solving equation (18) one obtains

$$\frac{T_c}{T_{c0}} = \frac{4}{\pi^2 \rho}\left[\psi\left(\frac{1}{2}+\frac{\rho}{2}\right) - \psi\left(\frac{1}{2}\right)\right], \tag{19}$$

where $\psi(x)$ is the di-gamma function and $\rho = \Gamma/\pi T_c$.

(a) $\Gamma \ll \pi T_c$ – one obtains

$$T_c \cong T_{c0}\left(1 - \frac{7\zeta(3)\Gamma}{\pi^3 T_{c0}}\right)\cdot \tag{20}$$

Note that $-dT_c/d\Gamma|_s \ll -dT_c/d\Gamma|_d = 2/\pi$, *i.e.* in the presence of the NS impurities the *s-wave FEP* pairing is more robust than the *d-wave FEP* pairing;

(b) $\Gamma \gg \pi T_c$ – in this case T_c goes to zero asymptotically, *i.e.*

$$T_c \approx \frac{\Gamma}{2\pi}\exp(-\pi\Gamma/4T_{c0}). \tag{21}$$

This means that the NS impurities are *pair-weakening* for the *s-wave FEP* pairing.

4.4 T_c for BCS s-wave pairing in the presence of FS impurities

Based on equation (1) one obtains T_c in the presence of the *FS* impurities

$$1 = V_{ep} T_c \sum_n \Theta(\Omega - |\omega_n|) Q(\omega_n, \Gamma_F). \tag{22}$$

One can show that $Q(\omega_n, \Gamma_F) = \pi/|\omega_n|$, which is the simple BCS formula in the absence of impurities. This means that the BCS *s-wave* pairing is unaffected by the *FS* impurities – the Anderson theorem holds.

5 Conclusions

In summary, it is shown here that: (a) by assuming that the pairing is due to the forward *E-P* scattering the critical temperature of clean systems T_{c0} depends linearly on the *E-P* coupling constant λ in the Migdal approximation; (b) the isotope effect is small in the weak coupling limit, *i.e.* $\alpha \ll 1$ for $T_{c0} \ll \Omega$; (c) impurities with the pronounced forward scattering (*FS* impurities) change analytical properties of the quasiparticle Green's function substantially and vertex corrections in the ladder approximation **screen** the single impurity Born scattering; (d) the *FS* impurities can push α to $1/2$ in the dirty limit ($\Gamma_F \gg T_c$); (e) the *FS* impurities affect in the same way *s-* and *d-wave FEP* pairing and they are pair-weakening for both pairings; (f) the NS isotropic impurities are pair-weakening for *s-wave FEP* pairing and pair-breaking for *d-wave FEP* pairing; (g) the *FS* impurities do not affect the usual BCS *s-wave* pairing.

Finally, we point out that the (non-Migdal) vertex corrections are important for the *FEP* pairing, because in the absence of momentum integration in the self-energy there is no small parameter in the theory, like $\lambda\omega_D/E_F$ in the Migdal theory. These effects can increase T_c as it has been asserted in [14]. However, it is shown that for the *FEP* pairing in the Migdal approximation T_c is linear function of the *E-P* coupling constant λ contrary to the exponential dependence obtained in [14]. The latter is due to the assumed small angle scattering in [14], which is different from the pronounced small **q** scattering studied here. The effect of the vertex corrections on T_c for the *FEP* pairing will be studied elsewhere.

In conclusion, the forward *E-P* and impurity scattering produce non-Fermi liquid effects in the normal state and non-BCS superconductivity what shall be studied in [20].

We thank Nils Schopohl, Evgenii G. Maksimov, Martin Peter, Dierk Rainer and Alexander I. Buzdin for discussions and support. This work was started during our stay at ISI, Torino, Italy, October 1996 and we acknowledge the ISI support.

References

1. A.P. Kampf, Phys. Rep. **249**, 219 (1994).
2. P. Monthoux, D. Pines, Phys. Rev. Lett. **69**, 961 (1992); Phys. Rev. B **47**, 6069 (1993); R.J. Radtke, K. Levin, H.B. Schüttler, M.R. Norman, Phys. Rev. B **48**, 15957 (1993).
3. P. Monthoux, D.J. Scalapino, Phys. Rev. Lett. **72**, 1874 (1994); S. Lenck, J.P. Carbotte, R.C. Dynes, Phys. Rev. B **49**, 9111 (1994).
4. S.V. Shulga, O.V. Dolgov, E.G. Maksimov, Physica C **178**, 266 (1991).
5. V.L. Ginzburg, E.G. Maksimov, Physica C **230-240**, 193 (1994).
6. O.V. Dolgov, S.V. Shulga, J. Supercond. **8**, 611 (1995).
7. S.I. Vedeneev, P. Samuely, S.V. Meshkov, G.M. Eliashberg, A.G.M. Jansen, P. Wyder, Physica C **198**, 47 (1992).
8. I. Bozovic, J.N. Eckstein, MRS Bulletin, Vol. XX, No. 5, May 1995; Physica C **235-240**, 178 (1994).
9. M. Kulić, R. Zeyher, Phys. Rev. B **49**, 4395 (1994); Physica C **199-200**, 358 (1994); Physica C **235-240**, 2151 (1994); Phys. Rev. B **53**, 2850 (1996); Phys. Rev. B **54**, 8985 (1996).
10. M. Grilli, C. Castellani, Phys. Rev. B **50**, 16880 (1994).
11. J. Keller, C.E. Leal, F. Forsthofer, Physica C **206-207**, 739 (1995).
12. A.A. Abrikosov, Physica C **244**, 243 (1995); J. Ruvalds *et al.*, Phys. Rev. B **51**, 3797 (1995).
13. M. Weger *et al.*, J. Low Temp. Phys. **95**, 131 (1994); G. Santi *et al.*, J. Supercond. **8**, 215 (1995).
14. L. Pietronero, S. Strässler, C. Grimaldi, Phys. Rev. B **52**, 10516 (1995); Phys. Rev. B **52**, 10530 (1995); P. Benedetti, C. Grimaldi, L. Pietronero, G. Varelogiannis, Europhys. Lett. **28**, 351 (1994).

15. P.W. Anderson, in *The Hubbard Model: Its Physics and Mathematical Physics*, edited by D. Baeriswyl *et al.* (Plenum Press, New York 1995), p. 217; G. Baskaran, *ibid.* p. 237.
16. G. Varelogiannis, E.N. Economou, Europhys. Lett. **42**, 313 (1998); G. Varelogiannis, L. Pietronero, Phys. Rev. B **52**, R15573 (1995); G. Varelogiannis *et al.*, Phys. Rev. B **54**, R6877 (1996).
17. V.A. Khodel, V.R. Shaginyan, JETP Lett. **51**, 553 (1990); P. Nozières, J. Phys. I France **2**, 443 (1992); J. Dukelsky, V.A. Khodel, P. Schuck, V.R. Shaginyan, Z. Phys. B **102**, 245 (1997).
18. V.N. Kostur, B. Mitrović, Phys. Rev. B **50**, 12774 (1994); Phys. Rev. B **51**, 6064 (1995).
19. A.I. Lichtenstein, M.L. Kulić, Physica C **245**, 186 (1995); T. Dahm, C.T. Rieck, L. Tewordt, S. Wermbter, Physica C **233**, 179 (1994).
20. O.V. Danylenko, O.V. Dolgov, M.L. Kulić, V. Oudovenko, in preparation.
21. P.B. Allen, B. Mitrović, in *Solid State Physics*, edited by H. Ehrenreich, F. Seitz, D. Turnbull (Academic Press, New York, 1982), Vol. 37, p. 1.
22. M.L. Kulić, Viktor Oudovenko, Solid State Comm. **104**, 375 (1997).